ATENISI UNIVERSITY
2005

Algebra 200 - Linear Algebra
Assignment 3

SOLUTION

Question 1:

(a). (i). Show that the set of vectors $(x_1, x_2, x_3)^T \in \mathbf{R}^3$ such that $x_1 + 2x_2 + 3x_3 = d$ is a subspace if and only if $d = 0$

Solution:

Let $S = \{(x_1, x_2, x_3)^T \in \mathbf{R}^3 : x_1 + 2x_2 + 3x_3 = 0\}$. Then

(A). $\vec{0} = (0,0,0) \in \mathbf{R}^3 \Rightarrow \vec{0} \in S$ since $0 + 2(0) + 3(0) = 0$

and

(B). Let $\vec{x} = (x_1, x_2, x_3)$, $\vec{y} = (y_1, y_2, y_3) \in S$ so that

$$x_1 + 2x_2 + 3x_3 = 0 \text{ and } y_1 + 2y_2 + 3y_3 = 0.$$

Adding the 2 equations we have

$$(x_1 + 2x_2 + 3x_3) + (y_1 + 2y_2 + 3y_3) = 0$$
$$\Rightarrow (x_1 + y_1) + (2x_2 + 2y_2) + (3x_3 + 3y_3) = 0$$
$$\Rightarrow (x_1 + y_1) + 2(x_2 + y_2) + 3(x_3 + y_3) = 0$$
$$\Rightarrow (x_1 + y_1, x_2 + y_2, x_3 + y_3) \in S$$
$$\Rightarrow (x_1, x_2, x_3) + (y_1, y_2, y_3) \in S$$
$$\Rightarrow \vec{x} + \vec{y} \in S.$$

(C). Let $c \in \mathbf{R}$, $\vec{x} \in S$ so that

$$c(x_1 + 2x_2 + 3x_3) = 0$$
$$c(0) = 0$$
$$\Rightarrow c\vec{x} \in S$$

Since the 3 conditions for a subspace is satisfied the S is a subspace. **5 Marks**

(ii). Show that the unit circle in $\mathbf{R}^2$: $\{(x_1, x_2)^T \in \mathbf{R}^2 : x_1^2 + x_2^2 = 1\}$ is not a subspace.

Solution:

Let $S = \{(x_1, x_2)^T \in \mathbf{R}^2 : x_1^2 + x_2^2 = 1\}$.

Now $\vec{0} = (0,0) \in \mathbf{R}^2 \Rightarrow \vec{0} \notin S$ since $0^2 + 0^2 \neq 1$. Hence S is not a subspace. **2 Marks**

(b). (i). Suppose $\mathbf{A} \in \mathbf{M}_{m,n}(\mathbf{R})$. Show that the set of $\mathbf{b}$ for which $\mathbf{A}x = \mathbf{b}$ has at least one solution is a subspace.

<u>Solution</u>:

Let $S = \{\bar{x} \in \mathbf{R}^n : A\bar{x} = \mathbf{b}\}$.

Suppose $\mathbf{b} = 0$ then $A\bar{x} = 0$.

And for $\bar{0} \in \mathbf{R}^n$, $A\bar{0} = 0 \Rightarrow \bar{0} \in S$.

Let $\bar{x}, \bar{y} \in S$ then $A(\bar{x} + \bar{y}) = A\bar{x} + A\bar{y} = 0 + 0 = 0 \Rightarrow (\bar{x} + \bar{y}) \in S$.

Let $\lambda \in \mathbf{R}$ so that $A(\lambda\bar{x}) = \lambda(A\bar{x}) = \lambda(0) = 0 \Rightarrow \lambda\bar{x} \in S$.

Hence S is a subspace for $\mathbf{b} = 0$. **4 Marks**

(ii). Let S denote the set $\{p \in P_3(\mathbf{C}) : p(1) = 0\}$. Show that S is a subspace of $P_3(\mathbf{C})$.

<u>Solution</u>:

Let $S = \{p \in P_3(\mathbf{C}) : p(1) = 0\}$.

Now since $0(x) = 0$, in particular $p(1) = 0$.

Let $p, q \in P_3(\mathbf{C})$ then $(p+q)(1) = p(1) + q(1) = 0 + 0 = 0 \Rightarrow p + q \in P_3(\mathbf{C})$.

And for $\lambda \in \mathbf{R}$, $\lambda p = (\lambda p)(1) = \lambda p(1) = \lambda(0) = 0 \Rightarrow \lambda p \in S$ **4 Marks**