ATENISI UNIVERSITY
2005

Calculus 100 L

Assignment 2 Due: 14th April 2005

Question 1.

Evaluate the following limits and explain carefully each step if

(i). $\lim_{x\to 1} \dfrac{x^2+3x+2}{2x^2-8}$ (ii). $\lim_{x\to 1} \dfrac{x^2-4x+3}{x^2-5x+4}$ (iii). $\lim_{x\to 1} \dfrac{x^{\frac{1}{3}}-1}{x-1}$ (iv). $\lim_{x\to 0} x^2 \sin\left(\dfrac{4}{x}\right)$

(v). $\lim_{x\to \infty} \dfrac{\sqrt{x^2+2}}{3x-6}$ (vi). $\lim_{x\to +\infty} \dfrac{4x^2+x-6}{5x^2-x+10}$ (vii). $\lim_{x\to 0} \dfrac{\sqrt{x+3}-\sqrt{3}}{x}$

Question 2.

We wish to show that $\lim_{x\to 0} \dfrac{1-\cos x}{x} = 0$ by doing the following steps:

(i). Let $f(x) = \dfrac{1-\cos x}{x}$. Show that $f(x) = -f(-x)$ for every $x \in \mathbf{R}$, $x \neq 0$.

(ii). Let x be such that $0 < x < \dfrac{\pi}{2}$. Draw a sketch of the following description; Let OAB be a right angled triangle formed by the points $O(0,0)$, $A(\cos x, 0)$ and $B(\cos x, \sin x)$. The angle OAB has value x in radians. Both points B and $C(1,0)$ on the circle or radius 1 centred at O. Then by using the fact that; the length of the arc $BC \geq$ the line segment BC;
show that

$$0 < \frac{1-\cos x}{x} < \cos\left(\frac{\pi - x}{2}\right).$$

(iii). Using both (i) and (ii), deduce that for every $x \in \mathbf{R}$ satisfying $0 < |x| < \dfrac{\pi}{2}$, we have

$$0 < |f(x)| < \cos\left(\frac{\pi - |x|}{2}\right).$$

(iv). Prove that $|f(x)| \to 0$ as $x \to 0$.

(v). Use the definition of limits to show that the result follows from (iv).

Question 3.

(i). Explain why the following calculation is incorrect.

$$\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{x^2}\right) = \lim_{x\to 0^+} \frac{1}{x} - \lim_{x\to 0^+} \frac{1}{x^2} = +\infty - (+\infty) = 0$$

(ii). Show that

$\lim_{x\to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{x^2}\right) = -\infty$ (iii). Let $r(x) = \dfrac{f(x)}{g(x)}$, $g(x) \neq 0$, be a rational function. Under what conditions is it true that

$$\lim_{x\to a} r(x) = r(a)$$

(iv). Find

$$\lim_{x\to +\infty} \frac{c_0 + c_1 x + c_2 x^2 + \ldots + c_n x_n}{d_0 + d_1 x + d_2 x^2 + \ldots + d_m x^m} = 0$$

[Hint: Your answer will depend on whether $m < n$, $m = n$ or $m > n$.]