

ATENISI UNIVERSITY
2005

Calculus 200 Assignment 1

Due: 18th March 2005

Answer 3 Questions ONLY!

Question 1:

- (a). Find a $\delta > 0$, for the $\lim_{x \rightarrow -1} (7x + 5) = -2$, where $\varepsilon = 0.01$.
- (b). Prove that $\lim_{x \rightarrow 3} (4x - 5) = 7$.
- (c). Show that $\lim_{h \rightarrow 0} \frac{1 - \cosh h}{h} = 0$.
- (d). Prove: if there are constants L, M such that $L \leq f(x) \leq M$ for all $x \in (a, b)$ and $0 \in (a, b)$, excepting the case when $x = 0$ where the inequality does not hold, then $\lim_{x \rightarrow 0} xf(x) = 0$.

Question 2:

- (a). Find $\frac{dy}{dx}$ if $y = \sinh^3(2x)$.
- (b). Evaluate $\int \sinh^6(x) \cosh(x) dx$
- (c). Evaluate $\int_{\ln 2}^{\ln(2\sqrt{5})} \frac{e^{-x} dx}{\sqrt{1 - e^{-2x}}}$
- (d). Use the reduction formula to evaluate $\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}} dx$. (*Hint: First make a substitution*)

Question 3:

- (a). Derive that $\int \sin^n(x) dx = -\frac{1}{n} \sin^{n-1}(x) \cos(x) + \frac{n-1}{n} \int \sin^{n-2}(x) dx$.
- (b). Show that $\int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx$ and hence evaluate $\int x^2 e^{3x} dx$ (*Hint: First make a substitution*).
- (c). Let f be a continuous function whose second derivative is continuous on $[-1, 1]$. Show that $\int_{-1}^1 xf''(x) dx = f'(1) + f'(-1) + f(-1) - f(1)$.
- (d). Use integration by parts on $\int \frac{1}{x} dx$ with $u = \frac{1}{x}$ and $dv = dx$. Explain.

Question 4:

- (a). Let $m, n \in \mathbf{Z}^+$, $m \neq n$. Prove that $\int_0^{2\pi} \sin(mx) \cos(nx) dx = 0$
- (b). Show that $\cosh^{-1}(x) = \ln(x + \sqrt{x^2 - 1})$ and hence find $\frac{d}{dx}(\cosh^{-1}(x))$
- (c). Evaluate $\int \frac{e^x}{\sqrt{1 + e^x + e^{2x}}} dx$
- (d). Find $\int \frac{3x^2 - x + 1}{x^3 - x^2} dx$

END